Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 11 June 2024

Afternoon (Time: 2 hours)

Paper reference **9MA0/02**

Mathematics

Advanced

PAPER 2: Pure Mathematics 2

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1.
$$y = 4x^3 - 7x^2 + 5x - 10$$

(a) Find in simplest form

(i) $\dfrac{dy}{dx}$

(ii) $\dfrac{d^2y}{dx^2}$

(3)

(b) Hence find the exact value of x when $\dfrac{d^2y}{dx^2} = 0$

(2)

a) $y = 4x^3 - 7x^2 + 5x - 10$

(i) $\dfrac{dy}{dx} = 12x^2 - 14x + 5$ 2

(ii) $\dfrac{d^2y}{dx^2} = 24x - 14$ 1

b) $\dfrac{d^2y}{dx^2} = 0$,

$24x - 14 = 0$ 1

+14 +14

$24x = 14$

÷24 ÷24

$x = \dfrac{14}{24}$ 1

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Question 1 continued

(Total for Question 1 is 5 marks)

2. Jamie takes out an interest-free loan of £8100

Jamie makes a payment every month to pay back the loan.

Jamie repays £400 in month 1, £390 in month 2, £380 in month 3, and so on, so that the amounts repaid each month form an arithmetic sequence.

(a) Show that Jamie repays £290 in month 12 **(1)**

After Jamie's Nth payment, the loan is completely paid back.

(b) Show that $N^2 - 81N + 1620 = 0$ **(2)**

(c) Hence find the value of N. **(2)**

common difference, $d = 390 - 400$
$= -10$

first term, $a = 400$

$T_n = a + (n-1)d$, where $n = 12$

a) Month 12, $T_{12} = 400 + (12-1) \times -10$
$= 400 + (-110)$
$= 290$ 1 $\therefore$ Jamie repays £290 in month 12

b) $S_n = \frac{n}{2}(2a + (n-1)d)$

$8100 = \frac{n}{2}(2(400) + (n-1) \times -10)$ 1

$8100 = \frac{n}{2}(800 - 10n + 10)$

$8100 = 400n - 5n^2 + 5n$

↓

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Question 2 continued

$8100 = 405n - 5n^2$

$5n^2 - 405n + 8100 = 0$

$\div 5$

$n^2 - 81n + 1620 = 0$ ① (shown)

c) $n^2 - 81n + 1620 = 0$

$(n-45)(n-36) = 0$

$n = 45$ or $n = 36$ ①

$n = 36$ (since Jamie has cleared his loan by 36^{th} payment) ①

(also after $n = 36$, the repayment starts becoming negative which is impossible)

(Total for Question 2 is 5 marks)

3. The point $P(3, -2)$ lies on the curve with equation $y = f(x)$, $x \in \mathbb{R}$

Find the coordinates of the point to which P is mapped when the curve with equation $y = f(x)$ is transformed to the curve with equation

(i) $y = f(x - 2)$

(ii) $y = f(2x)$

(iii) $y = 3f(-x) + 5$

(4)

$y = f(x)$

At point P : when $x = 3$, $f(3) = y = -2$

(i) $y = f(x-2)$

when $y = -2$, $(x-2) = 3$

$x = 5$

$\therefore (5, -2)$ 1

(ii) $y = f(2x)$

when $y = -2$, $(2x) = 3$

$x = 1.5$

$\therefore (1.5, -2)$ 1

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Question 3 continued

(iii) $y = 3f(-x) + 5$

the x-coordinate becomes negated

the y-coordinate is multiplied by 3 and has 5 added to it

Hence, $x = -(3) = -3$ (1)

$y = 3(-2) + 5 = -1$

$\therefore (-3, -1)$ (1)

(Total for Question 3 is 4 marks)

4. A sequence $u_1, u_2, u_3, \ldots$ is defined by

$$u_{n+1} = ku_n - 5$$
$$u_1 = 6$$

where k is a positive constant.

Given that $u_3 = -1$

(a) show that

$$6k^2 - 5k - 4 = 0$$

(2)

(b) Hence

(i) find the value of k,

(ii) find the value of $\sum_{r=1}^{3} u_r$

(3)

a) $u_2 = ku_1 - 5$

$u_1 = 6$

$u_2 = 6k - 5$

$u_3 = ku_2 - 5$

$u_3 = -1$ $\quad$ $u_2 = 6k - 5$

$-1 = k(6k-5) - 5$ 1

$-1 = 6k^2 - 5k - 5$

$6k^2 - 5k - 4 = 0$ (shown) 1

b) (i) $6k^2 - 5k - 4 = 0$

$(3k-4)(2k+1) = 0$

$k = \frac{4}{3}$ or $k = -\frac{1}{2}$

$k = \frac{4}{3}$ (since k is a positive constant) 1

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Question 4 continued

(ii) $\sum_{r=1}^{3} u_r = \overset{u_1}{6} + \underbrace{\frac{4}{3}(6) - 5}_{u_2} + \overset{u_3}{(-1)}$ (1)

$= 6 + 3 - 1$

$= 8$ (1)

(Total for Question 4 is 5 marks)

5. Given that θ is small and in radians, use the small angle approximations to find an approximate numerical value of

$$\frac{\theta \tan 2\theta}{1 - \cos 3\theta}$$

(3)

Small angle approximation :

$\sin\theta \approx \theta$

$\cos\theta = 1 - \frac{\theta^2}{2}$

$\tan\theta = \theta$

$$\frac{\theta \tan 2\theta}{1 - \cos 3\theta} = \frac{\theta(2\theta)}{1 - \left(1 - \frac{(3\theta)^2}{2}\right)}$$ 1 1

$$= \frac{2\theta^2}{\frac{9\theta^2}{2}}$$

$$= 2\theta^2 \times \frac{2}{9\theta^2}$$

$$= \frac{4\theta^2}{9\theta^2}$$

$$= \frac{4}{9}$$ 1

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Question 5 continued

(Total for Question 5 is 3 marks)

6.

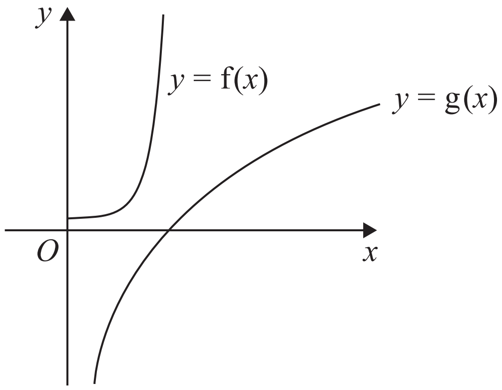

Figure 1

Figure 1 shows a sketch of the curves with equations $y = f(x)$ and $y = g(x)$ where

$$f(x) = e^{4x^2 - 1} \qquad x > 0$$

$$g(x) = 8\ln x \qquad x > 0$$

(a) Find

(i) $f'(x)$

(ii) $g'(x)$

(2)

Given that $f'(x) = g'(x)$ at $x = \alpha$

(b) show that α satisfies the equation

$$4x^2 + 2\ln x - 1 = 0$$

(2)

The iterative formula

$$x_{n+1} = \sqrt{\frac{1 - 2\ln x_n}{4}}$$

is used with $x_1 = 0.6$ to find an approximate value for α

(c) Calculate, giving each answer to 4 decimal places,

(i) the value of x_2

(ii) the value of α

(3)

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Question 6 continued

a) (i) $f(x) = e^{4x^2-1}$

$f'(x) = 8x \cdot e^{4x^2-1}$ 1

(ii) $g(x) = 8 \ln x$

$g'(x) = 8 \times \frac{1}{x} = \frac{8}{x}$ 1

b) Given $g'(x) = f'(x)$ at $x = a$

$8xe^{4x^2-1} = \frac{8}{x}$

$\div 8x$

$e^{4x^2-1} = \frac{8}{8x^2}$

$e^{4x^2-1} = \frac{1}{x^2}$

$\times \ln$

$4x^2-1 = \ln\left(\frac{1}{x^2}\right)$ 1

$4x^2-1 = \ln 1 - \ln x^2$

$4x^2-1 = -2\ln x$

$+2\ln x$

$4x^2 + 2\ln x - 1 = 0$ (shown)

1

Question 6 continued

c) $x_{n+1} = \sqrt{\dfrac{1 - 2\ln x_n}{4}}$, $x_1 = 0.6$

(i) $x_2 = \sqrt{\dfrac{1 - 2\ln 0.6}{4}}$ (1)

$= 0.7109$ (1)

(ii) $x_1 = 0.6$

$x_2 = 0.7109$

$x_3 = \sqrt{\dfrac{1 - 2\ln 0.7109}{4}} = 0.6485$

$x_4 = \sqrt{\dfrac{1 - 2\ln 0.6485}{4}} = 0.6830$

$x_5 = \sqrt{\dfrac{1 - 2\ln 0.6830}{4}} = 0.6638$

$x_6 = \sqrt{\dfrac{1 - 2\ln 0.6638}{4}} = 0.6745$

$x_7 = \sqrt{\dfrac{1 - 2\ln 0.6745}{4}} = 0.6685$

$x_8 = \sqrt{\dfrac{1 - 2\ln 0.6685}{4}} = 0.6718$

$x_9 = \sqrt{\dfrac{1 - 2\ln 0.6718}{4}} = 0.6700$

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Question 6 continued

$x_{10} = \sqrt{\frac{1-2\ln 0.6700}{4}} = 0.6710$

$x_{11} = \sqrt{\frac{1-2\ln 0.6710}{4}} = 0.6704$

$x_{12} = \sqrt{\frac{1-2\ln 0.6704}{4}} = 0.6708$

$x_{13} = \sqrt{\frac{1-2\ln 0.6708}{4}} = 0.6706$

$x_{14} = \sqrt{\frac{1-2\ln 0.6706}{4}} = 0.6707$

$x_{15} = \sqrt{\frac{1-2\ln 0.6707}{4}} = 0.6706$

$x_{16} = \sqrt{\frac{1-2\ln 0.6706}{4}} = 0.6706$

By doing iteration, the number will converge to a value.

$\therefore a = 0.6706$ (1)

ALSO TAKE NOTE THAT YOU DON'T NEED TO WRITE OUT ALL ITERATIONS BY HAND!

(Total for Question 6 is 7 marks)

7.

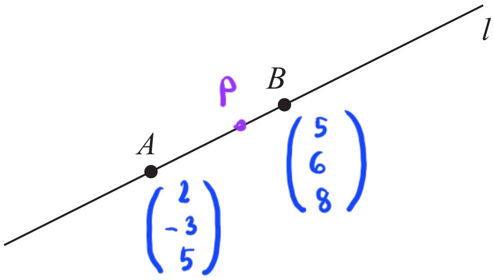

Figure 2

Figure 2 shows a sketch of the straight line l.

Line l passes through the points A and B.

Relative to a fixed origin O

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}$

(a) Find $\overrightarrow{AB}$

(1)

Given that a point P lies on l such that

$$|\overrightarrow{AP}| = 2|\overrightarrow{BP}|$$

(b) find the possible position vectors of P.

(4)

a) $\overrightarrow{OA} = \begin{pmatrix}2\\-3\\5\end{pmatrix}$ $\quad \overrightarrow{OB} = \begin{pmatrix}5\\6\\8\end{pmatrix}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\begin{pmatrix}2\\-3\\5\end{pmatrix} + \begin{pmatrix}5\\6\\8\end{pmatrix}$

$= \begin{pmatrix}3\\9\\3\end{pmatrix}$ 1

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Question 7 continued

b) $\overrightarrow{OP} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \overrightarrow{AP} = \begin{bmatrix} x-2 \\ y+3 \\ z-5 \end{bmatrix}, \quad \overrightarrow{BP} = \begin{bmatrix} x-5 \\ y-6 \\ z-8 \end{bmatrix}$

$|\overrightarrow{AP}| = 2|\overrightarrow{BP}| \Rightarrow \left|\begin{bmatrix} x-2 \\ y+3 \\ z-5 \end{bmatrix}\right| = 2\left|\begin{bmatrix} x-5 \\ y-6 \\ z-8 \end{bmatrix}\right|$ (1) $\Rightarrow \begin{matrix} (x-2)^2 = 4(x-5)^2 \\ (y+3)^2 = 4(y-6)^2 \\ (z-5)^2 = 4(z-8)^2 \end{matrix}$ (1)

$x: \quad x^2 - 4x + 4 = 4x^2 - 40x + 100$

$3x^2 - 36x + 96 = 0$

$x^2 - 12x + 32 = 0$

$(x-8)(x-4) = 0$

$x = 4, 8$ (1)

$z: \quad z^2 - 10z + 25 = 4z^2 - 64z + 256$

$3z^2 - 54z + 231 = 0$

$z^2 - 18z + 77 = 0$

$(z-7)(z-11) = 0$

$z = 7, 11$ (1)

$y: \quad y^2 + 6y + 9 = 4y^2 - 48y + 144$

$3y^2 - 54y + 135 = 0$

$y^2 - 18y + 45 = 0$

$(y-3)(y-15) = 0$

$y = 3, 15$

$\overrightarrow{OP} = \begin{bmatrix} 4 \\ 3 \\ 7 \end{bmatrix}, \begin{bmatrix} 8 \\ 15 \\ 11 \end{bmatrix}$ (1)

(Total for Question 7 is 5 marks)

8. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Prove that

$$\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1} \equiv 2\tan\theta\sec\theta \qquad \theta \neq (90n)^\circ,\ n \in \mathbb{Z}$$

(3)

(b) Hence solve, for $0 < x < 90°$, the equation

$$\frac{1}{\operatorname{cosec}2x - 1} + \frac{1}{\operatorname{cosec}2x + 1} = \cot 2x \sec 2x$$

Give each answer, in degrees, to one decimal place.

(4)

a) $\frac{1}{\operatorname{cosec}\theta - 1} + \frac{1}{\operatorname{cosec}\theta + 1} = \frac{\operatorname{cosec}\theta + 1 + \operatorname{cosec}\theta - 1}{(\operatorname{cosec}\theta - 1)(\operatorname{cosec}\theta + 1)}$ 1

$= \frac{2\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1}$

$= \frac{2\operatorname{cosec}\theta}{\cot^2\theta}$ 1

$= \frac{2}{\sin\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{2\sin\theta}{\cos^2\theta}$

$= \frac{2\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta}$

$= 2\tan\theta\sec\theta$ 1 (shown)

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Question 8 continued

b) from a), $2\tan 2x \sec 2x = \cot 2x \sec 2x$ (1)

$2\tan 2x \sec 2x - \cot 2x \sec 2x = 0$

$\sec 2x (2\tan 2x - \cot 2x) = 0$

$2\tan 2x - \cot 2x = 0$

$2\tan 2x = \cot 2x$

$2\tan 2x = \frac{1}{\tan 2x}$

$2\tan^2 2x = 1$

$\tan^2 2x = \frac{1}{2}$ (1)

$\tan 2x = \frac{1}{\pm\sqrt{2}}$ (1)

$2x = \tan^{-1}\frac{1}{\sqrt{2}}, \quad \tan^{-1}\frac{-1}{\sqrt{2}}$

$2x = 35.26^\circ, \; 180^\circ - 35.26^\circ$

$2x = 35.26^\circ, \; 144.74$ (for $0^\circ < 2x < 180^\circ$)

$x = 17.6^\circ, \; 72.4^\circ$ (1) (for $0^\circ < x < 90^\circ$)

Question 8 continued

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Question 8 continued

(Total for Question 8 is 7 marks)

9.

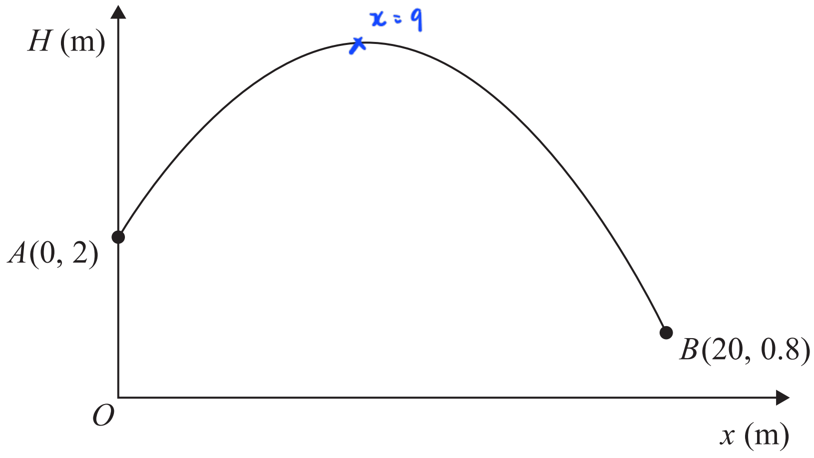

Figure 3

The graph in Figure 3 shows the path of a small ball.

The ball travels in a vertical plane above horizontal ground.

The ball is thrown from the point represented by A and caught at the point represented by B.

The height, H metres, of the ball above the ground has been plotted against the horizontal distance, x metres, measured from the point where the ball was thrown.

With respect to a fixed origin O, the point A has coordinates (0, 2) and the point B has coordinates (20, 0.8), as shown in Figure 3.

The ball reaches its maximum height when $x = 9$

A quadratic function, linking H with x, is used to model the path of the ball.

(a) Find H in terms of x.

(4)

(b) Give one limitation of the model.

(1)

Chandra is standing directly under the path of the ball at a point 16 m horizontally from O.

Chandra can catch the ball if the ball is less than 2.5 m above the ground.

(c) Use the model to determine if Chandra can catch the ball.

(2)

a) Model of the ball is a quadratic function.

Hence, use $ax^2 + bx + c = H$

For point $A(0,2)$: $a(0)^2 + b(0) + c = 2$

$\therefore c = 2$ 1

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Question 9 continued

$H = ax^2 + bx + 2$

Finding value of a and b :

when ball reach maximum height, $\frac{dH}{dx} = 0$ and $x = 9$

$\frac{dH}{dx} = 2ax + b$

$0 = 2(9)a + b$

$0 = 18a + b \Rightarrow b = -18a$ 1

At point B $(20, 0.8)$: $0.8 = a(20)^2 + (-18a)(20) + 2$

$0.8 = 400a - 360a + 2$ 1

$-1.2 = 40a$

$a = -0.03$

$b = -18(-0.03) = 0.54$

$\therefore H = -0.03x^2 + 0.54x + 2$ 1

b) Wind may affect the path of the ball. 1

c) when $x = 16$, $H = -0.03(16)^2 + 0.54(16) + 2$ 1

$= 2.96$ m $<$ 2.50 m

$\therefore$ Chandra would not be able to catch the ball.
1

Question 9 continued

Question 9 continued

(Total for Question 9 is 7 marks)

10.

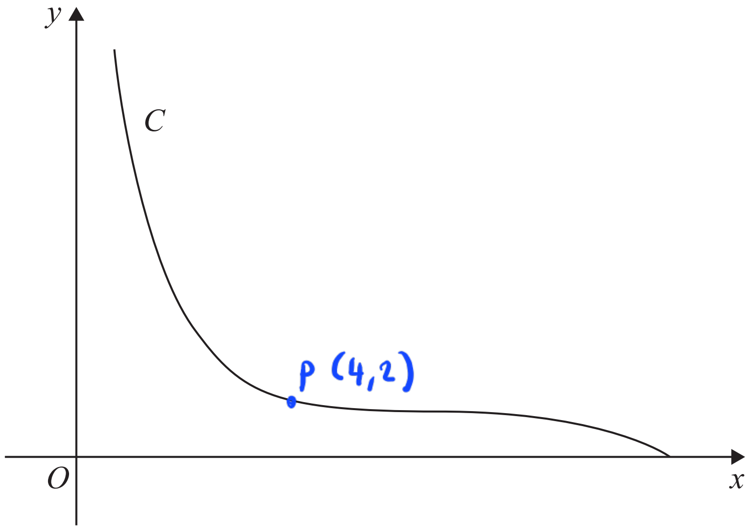

Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = (t+3)^2 \qquad y = 1 - t^3 \qquad -2 \leqslant t \leqslant 1$$

The point P with coordinates (4, 2) lies on C.

(a) Using parametric differentiation, show that the tangent to C at P has equation

$$3x + 4y = 20$$

(5)

The curve C is used to model the profile of a slide at a water park.

Units are in metres, with y being the height of the slide above water level.

(b) Find, according to the model, the greatest height of the slide above water level.

(1)

a) when $x = 4$ and $y = 2$,

$y = 1 - t^3 \Rightarrow 2 = 1 - t^3$

$t^3 = -1$

$t = -1$ (1)

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Question 10 continued

tangent of curve C at P

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

$$\frac{dy}{dt} = -3t^2 \quad , \quad \frac{dx}{dt} = 2(t+3)$$

$$\frac{dy}{dx} = -3t^2 \times \frac{1}{2(t+3)} \quad (1)$$

Substitute $t = -1$ into $\frac{dy}{dx}$

$$\frac{dy}{dx} = -3(-1)^2 \times \frac{1}{2(-1+3)}$$

$$= -3 \times \frac{1}{4}$$

$$\frac{dy}{dx} = -\frac{3}{4} \quad (1)$$

Equation of tangent C at P :

$$y - 2 = -\frac{3}{4}(x-4) \quad (1)$$

$$4y - 8 = -3x + 12$$

$$3x + 4y = 20 \quad \text{(shown)} \quad (1)$$

b) $y = 1 - t^3 \quad , \quad -2 \leqslant t \leqslant 1$

Slide is the highest when $t = -2$,

$$y = 1 - (-2)^3 = 1 - (-8)$$

$$= 9 \text{ m} \quad (1)$$

(Total for Question 10 is 6 marks)

11.

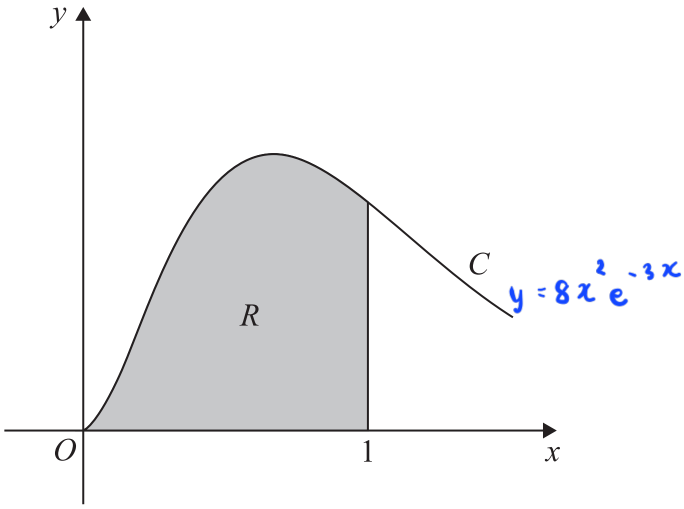

Figure 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 5 shows a sketch of part of the curve C with equation

$$y = 8x^2e^{-3x} \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 5, is bounded by

- the curve C
- the line with equation $x = 1$
- the x-axis

Find the exact area of R, giving your answer in the form

$$A + Be^{-3}$$

where A and B are rational numbers to be found.

(5)

$y = 8x^2 e^{-3x}$

$u = 8x^2$ $\quad \frac{du}{dx} = 16x$ $\quad \frac{dv}{dx} = e^{-3x}$ $\quad v = -\frac{1}{3}e^{-3x}$

Question 11 continued

$$\left[u = \frac{-16x}{3} \quad \frac{du}{dx} = \frac{-16}{3} \quad dv = e^{-3x}\,dx \quad v = \frac{-e^{-3x}}{3} \right]$$

$$\int y\,dx = \int 8x^2 e^{-3x}\,dx = -\frac{8x^2}{3} e^{-3x} \text{ (1)} - \int -\frac{16x}{3} e^{-3x}\,dx \text{ (1)}$$

$$= -\frac{8x^2}{3} e^{-3x} - \left[\frac{16x}{9} e^{-3x} - \int \frac{16}{9} e^{-3x}\,dx \right] \text{ (1)}$$

$$= -\frac{8x^2}{3} e^{-3x} - \left[\frac{16x}{9} e^{-3x} + \frac{16}{27} e^{-3x} \right]$$

$$= \left[-\frac{8x^2}{3} e^{-3x} - \frac{16x}{9} e^{-3x} - \frac{16}{27} e^{-3x} \right]_0^1 \text{ (1)}$$

$$= -\frac{8}{3} e^{-3} - \frac{16}{9} e^{-3} - \frac{16}{27} e^{-3} - \left(-\frac{16}{27} \right)$$

$$= \frac{16}{27} - \frac{136}{27} e^{-3} \text{ (1)}$$

Question 11 continued

Question 11 continued

(Total for Question 11 is 5 marks)

12. (a) Express $\dfrac{1}{V(25-V)}$ in partial fractions.

(2)

The volume, V microlitres, of a plant cell t hours after the plant is watered is modelled by the differential equation

$$\frac{dV}{dt} = \frac{1}{10}V(25-V)$$

The plant cell has an initial volume of 20 microlitres.

(b) Find, according to the model, the time taken, in minutes, for the volume of the plant cell to reach 24 microlitres.

(5)

(c) Show that

$$V = \frac{A}{e^{-kt} + B}$$

where A, B and k are constants to be found.

(3)

The model predicts that there is an upper limit, L microlitres, on the volume of the plant cell.

(d) Find the value of L, giving a reason for your answer.

(2)

a) $\dfrac{1}{V(25-V)} = \dfrac{A}{V} + \dfrac{B}{25-V}$

$1 = (25-V)A + B(V)$

when $V=0$, $1 = 25A$. $\therefore A = \dfrac{1}{25}$ 1

when $V = 25$, $1 = 25B$. $\therefore B = \dfrac{1}{25}$

$\therefore \dfrac{1}{V(25-V)} = \dfrac{1}{25V} + \dfrac{1}{25(25-V)}$ 1

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Question 12 continued

b) $\frac{dV}{dt} = \frac{1}{10} V(25-V)$

To find V, we integrate the equation of $\frac{dV}{dt}$:

$\int \frac{1}{V(25-V)} dV = \int \frac{1}{10} dt$

$\int \frac{1}{25V} + \frac{1}{25(25-V)} dV = \frac{1}{10}t + c$ (1)

$= \frac{1}{25}\ln V - \frac{1}{25}\ln(25-V) = \frac{1}{10}t + c$ (1)

Finding value of c :

Initially, $t=0$ and $V=20$

$\frac{1}{25}\ln 20 - \frac{1}{25}\ln 5 = c$

$\frac{1}{25}\ln\frac{20}{5} = c$

$\therefore c = \frac{1}{25}\ln 4$ (1)

Finding time, t when $V = 24$:

$\frac{1}{25}\ln 24 - \frac{1}{25}\ln(1) = \frac{1}{10}t + \frac{1}{25}\ln 4$ (1)

$\frac{1}{25}\ln 24 - \frac{1}{25}\ln 4 = \frac{t}{10}$

$\frac{1}{25}\ln 6 = \frac{t}{10}$

1 hr = 60 minutes

$t = \frac{10}{25}\ln 6 \times 60 = 43$ minutes (1)

Question 12 continued

(c) $\frac{1}{25}\ln V - \frac{1}{25}\ln(25-V) = \frac{1}{10}t + \frac{1}{25}\ln 4$

$\times 25$

$\ln V - \ln(25-V) = 2.5t + \ln 4$

$-\ln 4$

$\ln V - \ln(25-V) - \ln 4 = 2.5t$

$\ln \frac{V}{4(25-V)} = 2.5t$

e^x both sides

$\frac{V}{4(25-V)} = e^{2.5t}$ (1)

$V = 4e^{2.5t}(25-V)$ (1)

$V = 100e^{2.5t} - 4Ve^{2.5t}$

$V + 4Ve^{2.5t} = 100e^{2.5t}$

$V(1+4e^{2.5t}) = 100e^{2.5t}$

$V = \frac{100e^{2.5t}}{1+4e^{2.5t}}$

$V = \frac{100}{e^{-2.5t}+4}$ (1)

d) As $t \to \infty$, $e^{-2.5t} = 0$ (1)

$V = \frac{100}{4} = 25$ (1)

$\therefore$ 25 microlitres is the upper limit.

Question 12 continued

(Total for Question 12 is 12 marks)

13. The world human population, P billions, is modelled by the equation

$$P = ab^t$$

where a and b are constants and t is the number of years after 2004

Using the estimated population figures for the years from 2004 to 2007, a graph is plotted of $\log_{10}P$ against t.

The points lie approximately on a straight line with

- gradient 0.0054
- intercept 0.81 on the $\log_{10}P$ axis

(a) Estimate, to 3 decimal places, the value of a and the value of b.

(4)

In the context of the model,

(b) (i) interpret the value of the constant a,

(ii) interpret the value of the constant b.

(2)

(c) Use the model to estimate the world human population in 2030

(2)

(d) Comment on the reliability of the answer to part (c).

(1)

a) $P = ab^t$ equation in terms of $\log_{10}$

$\log_{10} P = \log_{10} a + \log_{10} b^t$

$\underbrace{\log_{10} P}_{y} = \underbrace{t}_{x}\, \underbrace{\log_{10} b}_{m} + \underbrace{\log_{10} a}_{c} \leftarrow y = mx + c$

$\therefore$ gradient $= \log_{10} b = 0.0054$ (1)

$b = 10^{0.0054} = 1.013$ (1)

$\therefore$ y-intercept $= \log_{10} a = 0.81$ (1)

$a = 10^{0.81} = 6.457$ (1)

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Question 13 continued

b) i) $P = ab^t$

when $t = 0$, $P = a$

a = world human populations in billions in 2004. (1)

ii) $b = 1.013$ represents the scale factor of yearly increase in the human population. (1)

c) in 2030, $t = 26$

$P = 6.457 \times 1.013^{26}$ (1)

$= 9$ billions (1)

d) Not reliable since the data used for the model only covered 2004 to 2007. Model might not be suitable to be used until 2030. (1)

Question 13 continued

Question 13 continued

(Total for Question 13 is 9 marks)

14. The circle C_1 has equation

$$x^2 + y^2 - 6x + 14y + 33 = 0$$

(a) Find

(i) the coordinates of the centre of C_1

(ii) the radius of C_1

(3)

A different circle C_2

- has centre with coordinates $(-6, -8)$
- has radius k, where k is a constant

Given that C_1 and C_2 intersect at 2 distinct points,

(b) find the range of values of k, writing your answer in set notation.

(5)

a) $x^2 + y^2 - 6x + 14y + 33 = 0$

Use completing the square method for both x and y :

(i) $(x-3)^2 + (y+7)^2 - 9 - 49 + 33 = 0$ 1

$(x-3)^2 + (y+7)^2 - 25 = 0$

centre of C_1 = $(3, -7)$ 1

(ii) $(x-3)^2 + (y+7)^2 = 25$

$(x-3)^2 + (y+7)^2 = 5^2$

radius of C_1 = 5 1

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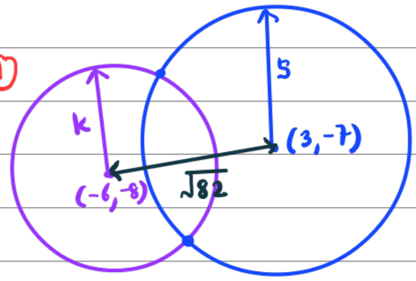
Question 14 continued

(b) distance between centers $= \sqrt{(3+6)^2 + (-7+8)^2}$ (1)

$= \sqrt{82}$ (1)



To make sure C_1 and C_2 intersect at 2 points :

$k > \sqrt{82} - 5$ (1) or $k < \sqrt{82} + 5$ (1)

$\therefore \sqrt{82} - 5 < k < \sqrt{82} + 5$ (1)

Question 14 continued

Question 14 continued

(Total for Question 14 is 8 marks)

15. The curve C has equation

$$(x + y)^3 = 3x^2 - 3y - 2$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y.

(5)

The point $P(1, 0)$ lies on C.

(b) Show that the normal to C at P has equation

$$y = -2x + 2$$

(2)

(c) Prove that the normal to C at P does **not** meet C again.

You should use algebra for your proof and make your reasoning clear.

(5)

a) $(x+y)^3 = 3x^2 - 3y - 2$

$(x^2+y^2+2xy)(x+y) = 3x^2-3y-2$

$(x^3+y^3+x^2y+y^2x+2x^2y+2y^2x) = 3x^2-3y-2$

$x^3+y^3+3x^2y+3y^2x = 3x^2-3y-2$

$3x^2+3y^2\left(\frac{dy}{dx}\right)$ ① $+3x^2\left(\frac{dy}{dx}\right)+6xy+6xy\left(\frac{dy}{dx}\right)$ ① $+3y^2 = 6x-3\frac{dy}{dx}$ ①

$(3y^2+3x^2+6xy+3)\frac{dy}{dx} = 6x-3x^2-6xy-3y^2$ ①

$$\frac{dy}{dx} = \frac{6x-3x^2-6xy-3y^2}{3y^2+3x^2+6xy+3}$$ ①

b) at point P (1,0).

$$\frac{dy}{dx} = \frac{6(1)-3(1)^2-6(1)(0)-3(0)^2}{3(0)^2+3(1)^2+6(1)(0)+3}$$

$= \frac{3}{6} = \frac{1}{2}$ (gradient of c)

∴ gradient of normal $= -2$ ①

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Question 15 continued

Equation of normal to c:

$y - 0 = -2(x-1)$

$y = -2x + 2$ (1)

(c) Substitute $y = -2x+2$ into $(x+y)^3 = 3x^2 - 3y - 2$ (1)

$(x - 2x + 2)^3 = 3x^2 - 3(-2x+2) - 2$

$(-x+2)^3 = 3x^2 + 6x - 6 - 2$

$-x^3 + 6x^2 - 12x + 8 = 3x^2 + 6x - 8$

$x^3 - 3x^2 + 18x - 16 = 0$ (1)

$$\begin{array}{r|l} & x^2 - 2x + 16 \\ x-1 & x^3 - 3x^2 + 18x - 16 \\ & -\ x^3 - x^2 \\ & -2x^2 + 18x \\ & -\ -2x^2 + 2x \\ & 16x - 16 \\ & -\ 16x - 16 \end{array}$$

Since $P(1,0)$, $(x-1)$ is known

$(x-1)(x^2 - 2x + 16) = 0$ (1)

For $x^2 - 2x + 16$, $b^2 - 4ac = (-2)^2 - 4(1)(16)$

$= 4 - 64$

$= -68 < 0$ (1)

Since $b^2 - 4ac < 0$, there are no real roots for $x^2 - 2x + 16$.

Hence, the normal does not meet at C again. (1)

Question 15 continued

Question 15 continued

Question 15 continued

(Total for Question 15 is 12 marks)

TOTAL FOR PAPER IS 100 MARKS